Numeracy in practice: teaching, learning and using mathematics

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Every child, every opportunity
Teacher knowledge and classroom culture have been factors frequently researched as contributors to effective teaching. Project Good Start (Thomson et al. 2005) found that effective teachers:

- have high expectations of all students and set challenging tasks and goals appropriate for each student
- integrate their content knowledge and their teaching skills to make connections that engage student interest and maintain involvement
- monitor student progress using their knowledge of each student’s current achievement and the next steps appropriate for them, and provide feedback to the student
- enjoy mathematics and take pleasure in students’ enjoyment and success.

The key elements emerging from studies examining effective numeracy teaching practices are a clear focus on concepts and thinking, an emphasis on valuing children’s strategies, and encouraging children to share their strategies and solutions (Thomson et al. 2005).

Teacher knowledge and scaffolding

The National Numeracy Review (COAG 2008) reports that ‘there is increasing agreement that the mathematical content knowledge required for teaching is connected to the teaching of particular content … and that how teachers hold knowledge may matter more than how much knowledge they hold’ (p. 66). This is clear reference to the importance of (mathematical) pedagogical content knowledge for effective teaching, and hence the role that teacher professional development must play in developing that kind of specialist knowledge.

Shulman (1987) distinguished three different types of knowledge needed for teaching:

- **Content knowledge** – knowledge of the mathematics being taught, the amount and organisation of the subject matter per se in the teacher’s mind.
- **Pedagogical knowledge** – knowledge of generic teaching strategies, such as questioning, grouping, planning, assessing, general factors that might impact learning.
- **Pedagogical content knowledge** – knowledge of the ways of representing and formulating the subject that makes it comprehensible to others, which includes knowledge of what makes the learning of specific topics easy or difficult, the conceptions and preconceptions that students of different ages and backgrounds bring with them. For mathematics teaching and its relation to numeracy achievement, teachers’ Mathematical Pedagogical Content Knowledge (MPCK) is cited as a key variable in many research studies.

Ma (1999) in her widely reported study of the differences between US and Chinese teachers pointed to four aspects of knowledge-for-teaching. These are:

- knowledge of basic mathematical ideas (i.e. the mathematical ideas that are pertinent to school mathematics)
• the ability to make connections between these ideas
• a capacity to create and use multiple representations of these ideas in teaching
• a deep knowledge of the curriculum continuum.

Her study found that while US teachers had generally taken more tertiary courses than their Chinese counterparts, they ‘displayed less subject matter knowledge and (less) pedagogical content knowledge’ (Ma 1999 cited in COAG 2008 p. 66). Her study underlines the importance of valuing the mathematical knowledge required for teaching in the primary and middle school years. This is not the same as a knowledge of advanced mathematics.

Teachers – at any level – who possess high-level mathematical knowledge have to work with children who clearly have more basic forms of that knowledge. A transmission model of teaching is not supported by research. Fennema and Romberg (1999) use the term ‘cognitively guided instruction’ that aims to understand and build on what the student is thinking. This places greater demands on knowledge-for-teaching since children’s responses and strategies can take the lesson in many possible directions. The teacher’s role is then to draw together those different directions with a clear focus on enhancing students’ understanding.

Researching Numeracy Teaching Approaches in Primary Schools (Department of Education, Science & Training 2004d) identifies a range of interaction patterns or scaffolding practices undertaken by the teacher (‘scaffolding’ because of the support they provide the student in their learning process until the student is ready to ‘stand on their own’).

The scaffolding practices can be selected and used appropriate to purpose, for example, to explore/make explicit what is known, challenge/extend students’ mathematical thinking, demonstrate the use of a mathematical instrument, or to assist students arrive at a key generalisation. In particular, they support teachers to make more informed decisions about how they will meet the learning needs of all students in the most appropriate way possible.

The twelve scaffolding practices that contribute to improved student learning outcomes are listed and described in table 5.1. When teachers used these scaffolding practices it had an effect on their own perceptions of what to teach and how to help students learn – that is, on their MPCK. As a result, there was a significant shift in what teachers perceived to be associated with effective mathematics teaching during the project. They changed from a predominant focus on activities, although these were still seen to be important, to recognition of the importance of teacher knowledge and the role of classroom culture (Department of Education, Science & Training 2004d).
Table 5.1: Scaffolding practices

<table>
<thead>
<tr>
<th><strong>Excavating</strong></th>
<th>Teacher systematically questions to find out what students know or to make the known explicit. Teacher explores children’s understanding in a systematic way</th>
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<tr>
<td>drawing out, digging, uncovering what is known, making it transparent</td>
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<tr>
<th><strong>Modelling</strong></th>
<th>Teacher shows students what to do and/or how to do it. Teacher instructs, explains, demonstrates, tells, offers behaviour for imitation</th>
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<td>demonstrating, directing, instructing, showing, telling, funnelling, naming, labelling, explaining</td>
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<tr>
<th><strong>Collaborating</strong></th>
<th>Teacher works interactively with students in-the-moment on a task to jointly achieve a solution. Teacher contributes ideas, tries things out, responds to suggestions of others, invites comments/opinions in what she/he is doing, accepts critique</th>
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<tr>
<td>acting as an accomplice, co-learner/problem-solver, co-conspirator, negotiating</td>
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<th><strong>Guiding</strong></th>
<th>Teacher observes, listens, monitors students as they work, asks questions designed to help them see connections, and/or articulate generalisations</th>
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<td>cuing, prompting, hinting, navigating, shepherding, encouraging, nudging</td>
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<tr>
<th><strong>Convince Me</strong></th>
<th>Teacher actively seeks evidence, encourages students to be more specific. Teacher may act as if he/she doesn’t understand what students are saying, encourages students to explain, to provide/obtain data</th>
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<tr>
<td>seeking explanation, justification, evidence; proving</td>
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<tr>
<th><strong>Noticing</strong></th>
<th>Teacher draws students attention to particular feature without telling students what to see/notice (i.e. by careful questioning, rephrasing or gestures), encourages students to question their sensory experience</th>
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<td>highlighting, drawing attention to, valuing, pointing to</td>
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<th><strong>Focusing</strong></th>
<th>Teacher focuses on a specific gap (i.e. a concept, skill or strategy) that students need to progress. Teacher maintains a joint collective focus and provides an opportunity for students to bridge the gap themselves</th>
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<tr>
<td>coaching, tutoring, mentoring, flagging, redirecting, re-voicing, filtering</td>
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<th><strong>Probing</strong></th>
<th>Teacher evaluates students understanding using a specific question/task designed to elicit a range of strategies, presses for clarification, identifies possible areas of need</th>
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<td>clarifying, monitoring, checking</td>
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<th><strong>Orienting</strong></th>
<th>Teacher sets the scene, poses a problem, establishes a context, invokes relevant prior knowledge and experience, provides a rationale (not necessarily at the beginning of the lesson, but at the beginning of a new task/idea)</th>
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<td>setting the scene, contextualising, reminding, alerting, recalling</td>
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<th><strong>Reflecting/Reviewing</strong></th>
<th>Teacher orchestrates a recount of what was learnt, a sharing of ideas and strategies. This typically occurs during whole class share time at the end of a lesson where learning is made explicit, key strategies are articulated, valued and recorded</th>
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<td>sharing, reflecting, recounting, summarising, capturing, reinforcing, reflecting, rehearsing</td>
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<th><strong>Extending</strong></th>
<th>Teacher sets significant challenge, uses open-ended questions to explore extent of children’s understanding, facilitate generalisations, provide a context for further learning</th>
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<td>challenging, spring boarding, linking, connecting</td>
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<th><strong>Apprenticing</strong></th>
<th>Teacher provides opportunities for more learned peers to operate in a student-as-teacher capacity, endorses student/student interaction</th>
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<td>inviting peer assistance, peer teaching, peer mentoring</td>
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Some related overseas and Australian research

One part of the Effective Teachers of Numeracy study (Askew et al. 1997) considered the classroom organisation of effective teachers; the second part considered teachers’ beliefs about teaching and mathematics. Effective teachers used a variety of classroom organisations, and while manifesting a range of different personal styles of teaching, it proved possible to discern three categories of approach to teaching:

- **Connectionist** teachers – who have beliefs and practices based on valuing children’s methods, using children’s understandings, and placing emphasis on making connections within mathematics.
- **Transmission** teachers – who have beliefs and practices based on the central role of teaching, and a view of mathematics as a collection of discrete skills, conventions and procedures to be taught and practised.
- **Discovery** teachers – who have beliefs based on the central role of learning, and a view of mathematics as being developed by children, particularly through interactions with concrete materials.

The connectionist teachers were found to be the most effective. However, when an attempt was made to replicate these findings in the Leverhulme Numeracy Research Programme, the results were less clear-cut (Brown 2000):

In the earlier study (Askew et al. 1997), there were clear-cut results in a small sample where relatively high mean gains were associated with teachers with connectionist orientations and relatively low mean gains with transmission and discovery orientations.

In the Leverhulme study we have found some of the same features as before in many of the teachers with high and low gains, but the picture is more complex, especially when trying to predict gains from features observed in lessons rather than teacher beliefs.

Nonetheless a connectionist orientation also appeared as a category for effective teachers in the Early Years Numeracy Project (ENRP). One of the aims of the ENRP (Clarke et al. 2001) was to gain insights into effective teachers and effective schools. Teachers whose students had shown ‘high growth’ were studied intensively through lesson observations, interviews and questionnaires. The distinguishing characteristics of effective teaching was classified into ten categories which can all be viewed as finer grained manifestation of pedagogical content knowledge:

- mathematical focus
- features of tasks
- materials, tools and representations
- adaptations/connections/links
- organisational style(s), teaching approaches
• learning community and classroom interaction
• expectations
• reflection
• assessment methods
• personal attributes of the teacher

These categories are described in Table 5.2.

Table 5.2: Effective teaching practices emerging from ENRP case studies

<table>
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<th>Effective early numeracy teachers…</th>
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| **Mathematical focus** | • focus on important mathematical ideas  
• make the mathematical focus clear to the children |
| **Features of tasks** | • structure purposeful tasks that enable different possibilities, strategies and products to emerge  
• choose tasks that engage children and maintain involvement |
| **Materials, tools and representations** | • use a range of materials/representations/contexts for the same concept |
| **Adaptions/connections/links** | • use teachable moments as they occur  
• make connections to mathematical ideas from previous lessons or experiences |
| **Organisational style(s), teaching approaches** | • engage and focus children’s mathematical thinking through an introductory, whole group activity  
• choose from a variety of individual and group structures and teacher roles within the major part of the lesson |
| **Learning community and classroom interaction** | • use a range of question types to probe and challenge children’s thinking and reasoning  
• hold back from telling children everything  
• encourage children to explain their mathematical thinking/ideas  
• encourage children to listen and evaluate others’ mathematical thinking/ideas, and help with methods and understanding  
• listen attentively to individual children  
• build on children’s mathematical ideas and strategies |
| **Expectations** | • have high but realistic mathematical expectations of all children  
• promote and value effort, persistence and concentration |
| **Reflection** | • draw out key mathematical ideas during and/or towards the end of the lesson  
• after the lesson, reflect on children’s responses and learning, together with activities and lesson content |
| **Assessment methods** | • collect data by observation and/or listening to children, taking notes as appropriate  
• use a variety of assessment methods  
• modify planning as a result of assessment |
| **Personal attributes of the teacher** | • believe that mathematics learning can and should be enjoyable  
• are confident in their own knowledge of mathematics at the level they are teaching  
• show pride and pleasure in individuals’ success |